Numerical Simulations of Driven Supersonic Relativistic MHD Turbulence

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Abstract. Models for GRB outflows invoke turbulence in relativistically hot magnetized fluids. In order to investigate these conditions we have performed high-resolution three-dimensional numerical simulations of relativistic magneto-hydrodynamical (RMHD) turbulence. We find that magnetic energy is amplified to several percent of the total energy density by turbulent twisting and folding of magnetic field lines. Values of $\varepsilon_B \gtrsim 0.01$ are thus naturally expected. We study the dependence of saturated magnetic field energy fraction as a function of Mach number and relativistic temperature. We then present power spectra of the turbulent kinetic and magnetic energies. We also present solenoidal (curl-like) and dilatational (divergence-like) power spectra of kinetic energy. We propose that relativistic effects introduce novel couplings between these spectral components. The case we explore in most detail is for equal amounts of thermal and rest mass energy, corresponding to conditions after collisions of shells with relative Lorentz factors of several. These conditions are relevant in models for internal shocks, for the late afterglow phase, for cocoon material along the edge of a relativistic jet as it propagates through a star, as well neutron stars merging with each other and with black hole companions.

We find that relativistic turbulence decays extremely quickly, on a sound crossing time of an eddy. Models invoking sustained relativistic turbulence to explain variability in GRB prompt emission are thus strongly disfavored unless a persistant driving of the turbulence is maintained for the duration of the prompt emission.

Keywords: gamma-rays: bursts - hydrodynamics:turbulence - methods:numerical - relativistic MHD
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INTRODUCTION

Turbulent cascades in astrophysics are invoked to explain a range of phenomena, including the modification of galactic magnetic fields [1] and star formation rates [2, 3, 4]. In this regime of gasdynamical turbulence it is safe to use the equations of isothermal magnetohydrodynamics, because the relevant flows are slow, cold, and weakly magnetized. However, magnetic fields may undergo turbulent amplification in environments where these assumptions are grossly violated. For example, the amplification of magnetic fields at neutron star mergers is driven by the Kelvin-Helmholtz instability [5, 6] operating in an extremely hot and strongly magnetized turbulent medium. Also, gamma-ray burst (GRB) outflows involve ultra-relativistic bulk flows containing mildly relativistic (warm) thermal velocities, [7, 8] and require weak magnetic fields to account for observed synchrotron radiation. Due to the additional coupling between inertial dynamics and gas pressure introduced by relativistic effects, existing paradigms for compressible MHD turbulence cannot be employed to understand these environments.

The mechanism for prompt emission of gamma-ray bursts (GRBs) is a major open problem in astrophysics. The internal shock model [9, 10, 11] has been widely applied to modeling GRB prompt emission. Recently however, [12] have suggested an alternative model in which variability in the GRB prompt emission is produced by relativistic fluctuations in the velocity field in the frame of the outflow. [13] claim that observations of GRB080319B rule out an internal shock model and suggest that the prompt emission is produced near the deceleration radius ($\sim 10^{17}$cm) by emitters with random Lorentz factors of ($\Gamma \sim 10$) in the comoving frame of the outflow. [14] have computed light curves from simplified models and conclude that the observed light curves can be obtained in a turbulence or "mini-jet" model if the emitter size and the bulk and random Lorentz factors satisfy tight constraints which may be inconsistent with other light curve features. In these models the light curve variability comes from the random Lorentz factor. On physical grounds, however, fluid turbulence should have random fluctuations of the velocity field on scales of the sound speed ($\Gamma \sim 1$) since supersonic flow will rapidly shock and dissipate. Detailed simulations of the dissipation of supersonic turbulence is therefor of interest in evaluating the plausibility of novel models for GRB emission.

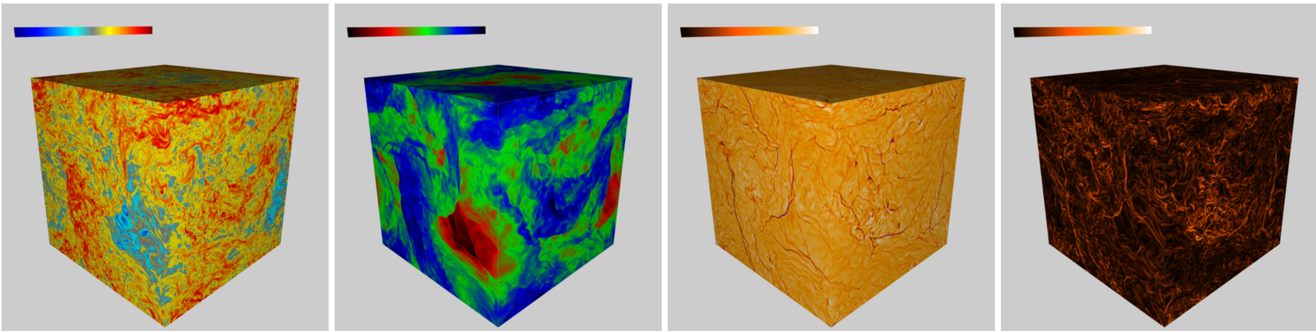

FIGURE 1. Shown are surface renderings of the magnetic energy (*top left*), gas pressure (*top right*), the vorticity (*bottom left*) and the divergence (*bottom right*) of the velocity field. This snapshot was taken after 2 light-crossing times, which is roughly the time at which ε_B becomes saturated. The volume averaged Mach number at this time is ~ 1.7. This model was run at resolution 512^3 using the forcing parameter $F_0 = 6.0$.

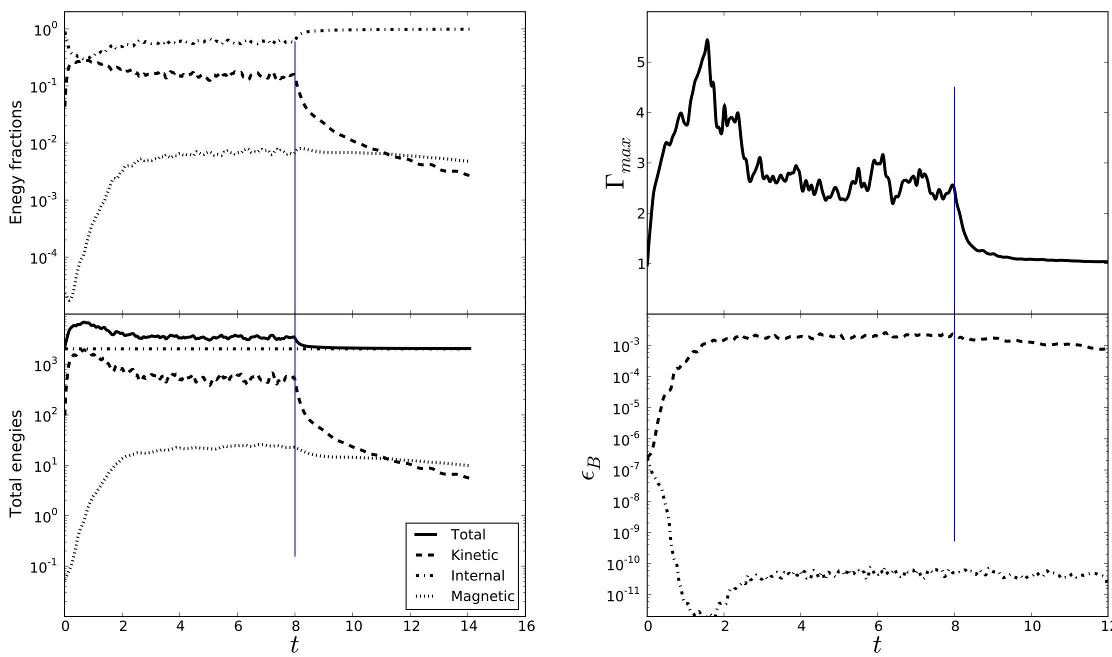

FIGURE 2. **Left**: Time histories of the total (*solid*), kinetic (*dashed*), inernal (*dash-dotted*), and magnetic (*dotted*) energies for supersonic turbulence. **Right**: Maximum Lorentz factor and the maximum (*dashed*) and average (*dash-dotted*) magnetic energy fraction. In these models, driving is switched off at $t = 8$ marked by the vertical line.

DESCRIPTION OF TURBULENCE MODELS

Our simulations are carried out on the 3-dimensional periodic cube $[-0.5, 0.5]^3$. All velocities are measured in units of c, and thus time is measured in units of the light-crossing time of the box. We initialize a uniform fluid of density $\rho_0 = 1.0$ having a background seed field $\mathbf{B}_0 = B_0\hat{\mathbf{x}}$ such that $\varepsilon_B = 10^{-5}$. The relativistic temperature, which we will define by $T \equiv P/\rho c^2$ is kept fixed throughout this study at $T = 1/3$. This value is deliberately chosen so that the fluid's rest mass energy density and internal energy density are in rough equipartition.

We use the same driving procedure in all runs, with the exception of the RMS forcing parameter F_0 which is adjusted to obtain the desired Mach number. We have adopted the forcing parameters $F_0 = 0.25$ and $F_0 = 6.0$ for

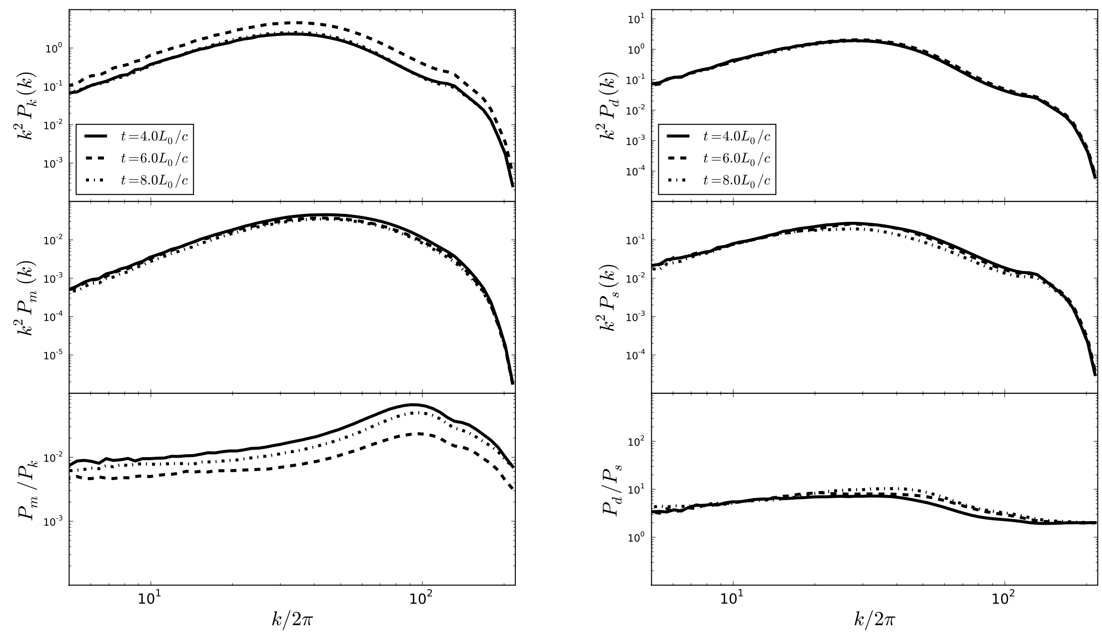

FIGURE 3. Shown are spherically integrated compensated power spectra of the kinetic and magnetic energy (**left**) and the solenoidal and dilatational velocity field modes (**right**) in a supersonic model having $\mathscr{M} \sim 1.7$ at 4 (*solid*), 6 (*dashed*), and 8 (*dashed-dotted*) light-crossing times.

subsonic and supersonic models respectively. Driving is switched on at the start of the simulation and kept on for 8 light-crossing times, after which driving is switched off and the flow is left to decay. For our supersonic turbulence model, we explore two branches of decay in the absence of driving, both starting from the same conditions at the moment driving is switched off. In the first branch, the cooling procedure is kept in place, maintaining the effectively isothermal euquation of state. In the second branch, we explore the free decay of adiabatic turbulence. Note that the adiabatic equation of state becomes practical again for decaying turbulence since the injection of energy has ceased, and thus the increase of pressure due to viscous heating at the grid scale is limited by the conservation of total energy.

TIME HISTORIES AND POWER SPECTRA

Figure 2 shows the fraction of energy in kinetic, internal, and magnetic parts from the moment driving is switched on through saturation and the decay phase. Not shown are the RMS velocities and Mach number. Within one light-crossing time after driving is switched on the flow velocities overshoot their steady state values. By three light-crossing times, they have fallen and the flow has reached a statistically steady state. Note that the early time transient behavior lasts only until turbulent kinetic energy has reached the grid scale. The transient may be explained by the absence of the effective viscosity imposed by smaller scale flow structures. As the energy cascade reaches smaller scales, this effective viscosity damps the motion of larger scale eddies, bringing the volume-averaged RMS velocities down. However, the total kinetic energy does not express the same transient feature and instead grows monotonically. As kinetic energy populates higher wave-numbers, the velocity dispersion becomes broader, much like an ensemble of gas particles being thermalized. However, the velocity structures present at these small scales are moving less rapidly than the larger eddies in which they are embedded.

We find that the maximum Lorentz factor of the flow decays exponentially on a time scale of order the domain's light-crossing time, which is also large eddy turnover time. The observation that an eddy dissipates its energy to smaller scales in roughly its own turnover time is in agreement with the conventional understanding. In particular, it confirms that transitory energy injection at a given length scale cannot sustain long-lived turbulent eddies at that scale. This may

also be observed in Figure 2 which shows the rapid decay of kinetic energy after driving is switched off. On the other hand the volume averaged magnetic energy fraction is sustained in the absence of driving.

Figure 3 shows spherically integrated power spectra of the kinetic and magnetic energy, and of the energy in solenoidal and dilatational velocity field modes. We have computed compenstated power spectra similar to [15], dividing by a power law for ease of interpretation. We have chosen for simplicity the power law k^{-2}, so that power spectra behaving as a power law with index -2 will appear horizontally.

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